Construction of control charts by using Fuzzy Multinomial -FM and EWMA Chart "Comparative study"

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Abstract:

The control chart is a graphical tool for monitoring the activities of a manufacturing process. It plays a vital role in Statistical Process Control (SPC). These charts help us to take correct decisions. The use of statistical methods in statistical quality control has a long history. The P-chart plays the important role in controlling the fraction of nonconforming article produced. The use of fuzzy has an effective role with inaccurate data. Great efforts have been made in the direction of combining the Quality Control with fuzzy concept. Therefore; Fuzzy control chart are connecting link between fuzzy logic and Quality Control technique, the goal of this connection is to combine the advantage of both disciplines in order to process deal with linguistic concepts and classified each item in more than two categories such as bad, medium, good, very good and excellent.

This paper presents an extension of standard control chart to deal with linguistic categories and the variable sampling size (VSS), it is named as fuzzy multinomial charts (FM-chart), and we illustrate this approach by numerical example. This paper is comparing FM-chart with the conventional p – chart and EWMA Control Chart. It is seen that FM chart with VSS performs better than the conventional charts, this method is more sensitive, accurate and more economic for assisting decision maker to control the operation system as early time, especially when there is a change in sample sizes.

Keywords: Quality Control- Process, FM chart, P- chart, variable sampling size, linguistic terms and Fuzzy control charts and EWMA-chart.

I. Introduction

SPC is considered as a scientific and efficient quality control systems in mass production. SPC control charts are used as the main tool to control the process. Has been the development of this innovative concept in 1920 by Dr. Walter A. Shewhart of Bell Labs phone and then many developments have taken place. Quality control is the process used to ensure a certain level of quality in the product or service. The primary objective of quality control is to ensure those products, services or processes provided meet specific requirements and can be relied up on. In 1924, Walter Shewhart designed the first control chart as follows:

Let w be a sample statistic that measures some quality characteristic of interest and suppose that the mean of w is μ_w and the standard deviation of w is σ_w. Then the center line (CL), the upper control limit (UCL) and the lower control limit (LCL) are defined as:

$$UCL = \mu_w + d\,\sigma_w$$
$$CL = \mu_w$$

$$UCL = \mu_w - d\,\sigma_w \qquad (1)$$

where d is the "distance" of the control limits from the center line, expressed in standard deviation units.

Zadeh in 1965, introduced the notion of fuzzy sets. After that, there have been efforts to apply it in statistics. When products are classified into mutually exclusive linguistic categories, fuzzy control charts are used such as Fuzzy multinomial control charts.

II. Fuzzy logic and Linguistic variable:

The concept of fuzzy logic plays a fundamental role in formulating quantitative fuzzy variables. These are the variables whose states are fuzzy members. The members represent linguistic concepts, such as very small, small, medium and so on, as interpreted in a particular context. The resulting constructs are usually called linguistic variables.

The fuzzy set theory has been applied in many fields, such as operations research, control theory and management studies, etc. Kangawaet al. [1993], Franceschini and Romano [1999], Gulby et al. [2004], Gulby and Kahraman [2006], have developed control charts based on linguistic data to monitor the ongoing process. Raz and Wang [1990] have proposed two approaches, namely probabilistic and membership, for the construction of control charts with linguistic data. In the probabilistic approach, the representative values of the linguistic data are obtained through medians, mid ranges, modes or the fuzzy averages of their membership functions.

By using the representative values, the control charts have been constructed with the applications of statistical methods. On the other hand, in the membership approach, the process level is taken as the centre line of the control chart and the control limits are constructed with fuzzy quantities.

In this paper, a fuzzy multinomial control chart (FM chart) has been proposed for linguistic variables (linguistic data set) with variable sample size are chosen randomly from a pre–determined set. Numerical application illustration is used to compare the proposed FM-chart with the standard p- chart and EWMA-Chart. The FM –chart with VSS is found to be more effective for identifying the shift in the process mean and more sensitive, accurate and economically for assisting decision maker to control the operation system as early time, especially when there is a change in sample sizes.

III. Methodology

In this paper a fuzzy multinomial control chart (FM chart) for linguistic variables with variable sample size is used and compare with the standard p – chart and EWMA-Chart of VSS. The FM – chart deals with a linguistic variable which is classified into more than two categories. Based upon Fuzzy set theory, a linguistic variable $(\tilde{L})$ is characterized by the set of k mutually exclusive members $(l_1, l_2, l_3, \dots, l_k)$. Attach a weight (m_i) to each term (l_i) that reflects the degree of membership in the set. Then it can be re-writes linguistic variable by a fuzzy set as:

$$\tilde{L} = \{(l_1, m_1), (l_2, m_2), \dots, (l_k, m_k)\} \qquad (2)$$

To monitor the production process, take independent samples of different sizes. The size of the sample to be drawn each time is decided by choosing a member randomly from $\{\mathrm{n}_1, \mathrm{n}_2, \dots, \mathrm{n}_s\}$.

IV. Fuzzy Multinomial Control Chart [6][10][11][12]

Control charts with variable sample sizes can be used for certain advantages. Costa [1994] has proposed a chart with variable sample sizes to study the shift in process mean. The same concept is extended for Multinomial control chart with variable sample sizes dealing with a linguistic variable. The statistical principles underlying the fuzzy multinomial control chart (FM - chart) with variable sample size are based on the multinomial distribution.

As defined in equation (2), $\tilde{L}$ is a linguistic set of variables which can take k mutually exclusive members $\{l_1, l_2, \ldots, l_k\}$. Assume that the production process is operating in a stable manner and p_i is the probability that an item is l_i ,$i = 1, 2 \ldots k$, and successive items produced are independent. Suppose that a random sample of size n_r units of the product is selected and let X_i ,$i = 1, 2 \ldots k$, be the number of items of the product that are l_i ,$i = 1, 2, \ldots, k$. Then $\{X_1, X_2 \ldots X_k\}$ has a multinomial distribution with parameters n_r and $p_1, p_2 \ldots, p_k$. It is known that each X_i ,$i = 1, 2 \ldots k$ marginally has a binomial distribution (i.e.) $X_i \sim Binomail\ (n_r, p_i)$, and:

$$E(X_i) = n_r \ p_i \qquad (3)$$

$$V(X_i) = n_r \ p_i(1 - p_i) \qquad (4)$$

The weighted average of the linguistic variable $\tilde{L}$ with sample size n_r is defined by:

$$\bar{\tilde{L}} = \frac{\sum_{i=1}^{k} X_i m_i}{\sum_{i=1}^{k} X_i} = \frac{\sum_{i=1}^{k} X_i m_i}{n_r} \qquad (5)$$

$$n_r \in \{n_1, n_2, \ldots, n_s\}$$

The control limits for FM – chart are:

$$UCL = E\left(\bar{\tilde{L}}\right) + d\sqrt{var\left(\bar{\tilde{L}}\right)}$$

$$CL = E\left(\bar{\tilde{L}}\right)$$

$$LCL = E\left(\bar{\tilde{L}}\right) - d\sqrt{var(\bar{\tilde{L}})} \qquad (6)$$

Where d is the distance of the control limits from the center line. By using equation (5) we obtain:

$$E\left(\bar{\tilde{L}}\right) = \sum_{i=1}^{k} p_i m_i \qquad (7)$$

$$var\left(\bar{\tilde{L}}\right) = \frac{1}{n_r}\left[\sum_{i=1}^{k} m_i^2 p_i(1 - p_i) - 2\sum_{\substack{i=1 \\ i<j}}^{k} \sum_{j=1}^{k} m_i m_j p_i p_j\right] \qquad (8)$$

$$n_r \in \{n_1, n_2, \ldots, n_s\}$$

Where $n_1, n_2, \ldots, n_s$ are pre-determined sample sizes.

A. Remark

- The results for $E\left(\bar{\tilde{L}}\right)$ and $var\left(\bar{\tilde{L}}\right)$ become the same as that of the results due to Amirzadeh [1] et al. [2008], when the sample size is fixed.
- If $\tilde{L} = \{(l_1, 0), (l_2, 1)\}$ is a linguistic variable, then FM-chart reduces to a p-chart with $p = p_r$ (an item is l_1), and the Upper and Lower fraction nonconforming chart are:

$$UCL_p = \bar{P} + 3\sqrt{\frac{\bar{P}(1 - \bar{P})}{n}}$$

$$CL = \bar{P}$$

$$LCL_p = \bar{P} - 3\sqrt{\frac{\bar{P}(1 - \bar{P})}{n}}$$

Depending on the value of (P) and (n) sometimes the value of (LCL) is less than 0, in this cases, we customarily set $LCL = 0$ and assume that the control chart only has an upper control limit

V. Exponentially Weighted Moving Average

The exponentially weighted moving average (EWMA) is a statistic for monitoring the process that averages the data in a way that gives less and less weight to data as they are further moved in time. The EWMA control chart is also a good alternative to the Shewhart control chart when we are interested in detecting small shifts, The performance of the EWMA control chart is approximately equivalent to that of the cumulative sum chart, and in some ways it is easier to set up and operate.

$$Z_i = \lambda X_i + (1 - \lambda)Z_{i-1} \qquad (9)$$

$$i = 1,2, \ldots, n$$

Where :

Z_i: is the mean of historical data(target value)

X_i: is the observation

n: is the number of observations to be monitored including EWMA

λ: is a suitable constant, such that $0 < \lambda \leq 1$ is a suitable that determines the depth of memory of the EWMA and $Z_0 = \mu$.

The sequence of value $Z_i, i = 0, 1, 2, \ldots, n$ is called exponential weighted moving average.

In general the Upper and Lower Control limits of EWMA chart are:

$$UCL = Z_0 + \ell\sigma\sqrt{\frac{\lambda}{2-\lambda}}$$

$$CL = Z_0$$

$$LCL = Z_0 + \ell\sigma\sqrt{\frac{\lambda}{2-\lambda}} \qquad (10)$$

Where ℓ is suitable in control with limit, and σ is the process standard deviation.

VI. Numerical Example:

On education line, the Mathematics is a subject of vital importance which underpins many activities of modern society, subject of Mathematics is important quality characteristic that has to be monitored Style of teaching in all college departments. Therefore; we took students degree of examination in the final examination (first round) for mathematics at the University of Sulaimani- Faculty of Administration and Economics for 26 years of different sizes are selected. The data was obtained from the Student Services database at the Faculty of Administration and Economics. Degree system may be classified by an expert team as fail, pass, Fair, Good, and Very Good, to monitor the quality of teaching process. Case of fail (student), is considered as a defective unit

The degrees of membership for the above assessment are taken as (1, 0.75, 0.5, 0.25 and 0) respectively.

Tables are numbered with Roman numbers.

Table.1: The number of student and the P-Charts Method for the 26 samples.

No.	n_r	Fail m=1	Pass m=0.75	Medium m=0.5	Good m=0.25	V.Good m=0	*P*	*UCLp*	*LCLp*
1	53	5	12	10	12	14	0.094	0.307	0.007
2	50	13	17	7	9	4	0.260	0.312	0.003
3	50	13	17	7	8	5	0.260	0.312	0.003
4	51	4	9	11	18	9	0.078	0.310	0.004
5	45	2	4	2	2	35	0.044	0.320	-0.005
.	.	.	.	.	.	.	.	.	.
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.	.	.	.	.	.	.	.	.	.
20	87	7	17	23	21	19	0.080	0.275	0.040
21	72	14	6	22	15	15	0.194	0.286	0.029
22	96	33	20	27	11	5	0.344	0.269	0.046
23	86	2	2	6	16	60	0.023	0.275	0.040
24	93	0	1	8	26	58	0.000	0.271	0.044
25	86	3	10	26	23	24	0.035	0.275	0.040
26	102	10	33	24	12	23	0.098	0.266	0.049

Table (1) contain the number of student, and the result of the examination in mathematics, for sample size (26) in Faculty of Administration and Economics.

And also the table (1) contains the UCL and LCL –chart, The chart given below depicts the conventional p – chart for 26 samples as shows in Fig.(1)

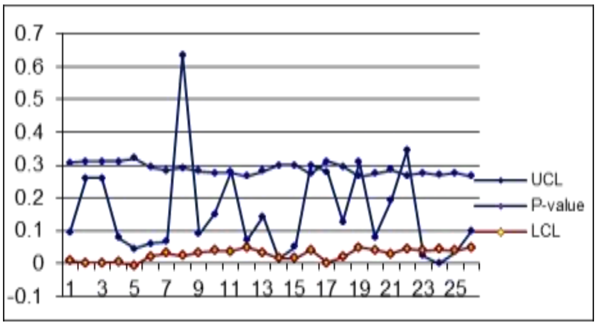

Fig.1 p – chart for the 26 samples.

In Figure 1, the first out of control signal is seen corresponding to 8th sample, for which the sample size is 66. From the p – chart, out of 1921 sample observations, 456 sample observations were needed to get the signal. The corresponding centre line and control limits are as under CL = 0.157, UCL = 0.25711 and LCL = 0.0577.

To construct the FM – chart the data with $\bar{L}$ and $\hat{p}_i$ are given in Table (2).

Table.2: Calculation of Mean, Variance of $\tilde{L}_i$ and control limits for various Sample size

No.	n_r	$\bar{\tilde{L}}$	$var(\bar{\tilde{L}})$	$E(\bar{\tilde{L}})$	UCL	LCL	EWMA
1	53	0.415	0.002	0.415	0.583	0.291	0.433
2	50	0.630	0.003	0.630	0.588	0.286	0.472
3	50	0.625	0.003	0.625	0.588	0.286	0.503
4	51	0.407	0.002	0.407	0.586	0.288	0.484
5	45	0.144	0.003	0.144	0.596	0.278	0.416
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
20	87	0.420	0.001	0.420	0.551	0.323	0.482
21	72	0.462	0.002	0.462	0.562	0.312	0.478
22	96	0.669	0.001	0.669	0.546	0.328	0.516
23	86	0.122	0.001	0.122	0.552	0.322	0.437
24	93	0.121	0.001	0.121	0.547	0.327	0.374
25	86	0.340	0.001	0.340	0.552	0.322	0.367
26	102	0.488	0.001	0.488	0.542	0.332	0.391

By using the equations (5), (6), and (7) and data in table(1) we calculating the determent the $\bar{L}$, and $var(\bar{L})$ as show in table (2), and to calculate the Upper and Lower Control limit for each sample size as shows in table (2), the main Lower and Upper control limit are (0.3399, 0.534) respectively and the central limit is (0.4371),as shows in fig(2).

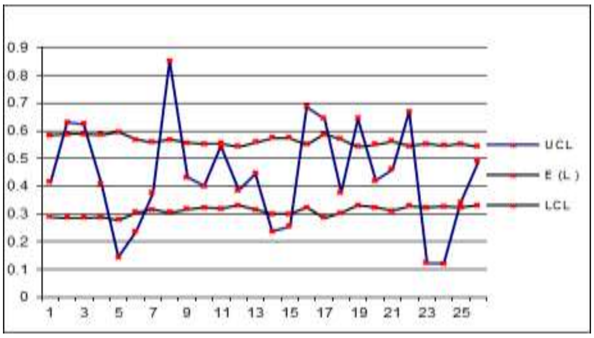

Fig.2 The FM – chart for the 26 samples.

Figure 2 shows that the process is out of control at samples 2,3,5,6, and 8, the respective sample sizes are 50, 50, 45, 65 and 66.

From figure 2, it is seen that the FM chart gives the first signal corresponding to 2nd sample. Whereas, in p – chart the first signal for the existence of assignable causes is seen only at the 8th sample. That is, in the case of FM – chart; only 103 samples are inspected to get the first out of control signal. But, 456 samples are to be inspected to get the alarm with the help of a P– chart. Thus, the FM is more economical and more sensitive in identifying any shift in the specified quality level.

To compare these procedures with exponentially weighted moving average, EWMA-chart are calculated as given in the table (3), and Fig. (3) shows that the EWMA-chart for data it seen the all samples in control, the LCL and UCL are (0.2651, 0.609) and the CL=0.437.

It seen that LCL of EWMA-chart is greater than LCL of FM-chart, which is equal to (0.5342277)

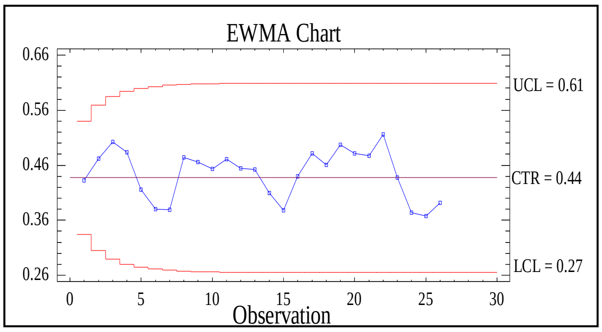

Fig.3 The EWMA – chart for the 26 samples.

This result without the level of ambition, which requires work to raise the level of scientific performance of this Faculty through the adoption of a methodology for the control charts accurate and scientific methods

VII. Conclusion:

The method of fuzzy logic seems more objective in this research

1. FM – chart has been proposed for linguistic data set is applicable on traditional variable control charts. The FM-chart has been compared with the conventional p–chart and EWMA- chart with VSS. It is shown that the FM – chart is more economical and more sensitive in giving the alarm for shift in the specified quality level. Use FM- charts with variable sample size are an efficient way and suitable for some deviations that lead to a change in sample sizes, as well as it can provide more flexibility for controlling process and have more appropriate mathematical description frame than control chart approach and give more meaning results than traditional quality control charts. Therefore; the proposed method is useful to study the attribute data with linguistic categories. The control limits are also developed based on Multinomial distribution with VSS.
2. FM-chart do not need to the large number of samples and therefore it does not need time to observe the course of the production process and needs to be little effort in planning for inspections and determine the results, so it is therefore useful because it helps in making decisions quickly (i.e. The FM – chart based on VSS reduce the cost of inspection).

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